BASIS OF AN ALGORITHM FOR ACCURATE CALCULATION OF TRANSIENT FERROMAGNETIC HYSTERESIS

Ing. Jean Riubrugent   Ing. Raúl Bianchi Lastra   Ing. Julieta Vernieri
Ing. María B. Barbieri
IITREE - Instituto de Investigaciones Tecnológicas para Redes y Equipos Eléctricos
UNIVERSIDAD NACIONAL DE LA PLATA
48 y 116 (1900) La Plata - ARGENTINA

1.- Introduction

It is sometimes very important, for an accurate simulation of complex transient processes which arise in electrical power systems, to model in a faithful way the ferromagnetic behaviour of equipment such as transformers or saturable compensating reactors.

Although computer programs for digital simulation of transients, such as EMTP, often include options for good modelling of saturation (the classic Type 98 and Type 93 of EMTP) the hysteresis effects have a rather "rough" representation based on the limit hysteresis loop (Type 96 of EMTP) [1].

A theorical model which is rich in insight about the transient behaviour of ferromagnetism is the one described in [2].

But an algorithm based on this model appears to be cumbersome to couple to a general transients programme such as EMTP [4].

In this paper, we develop another approach which, although along a thinking line similar to that of Biorci and Pescetti, gives a mathematical model simpler to translate into a useful algorithm and yet shows an overall behaviour which represents very well that of ferromagnetism during complex transients.

The proposed model takes as basic data of the material the usual limit hysteresis cycle (the same as given for Type 96 of EMTP) and, in addition, the so called curve of reversible permeability.

The information contained in these characteristic curves of the macroscopic behaviour of the material, is re-interpreted in terms of a "microscopic" conceptual model, and transformed into two implicit functions which fully describe the instantaneous state of the "inside structure" of the material, reflecting its complete past history.

Thereby, it is possible to develop a well structured and simple algorithm which follows, step by step, the state of the material and hence its macroscopic response, whatever the transient process in which it is involved.

The paper describes the proposed conceptual model, explains the theoretical background, develops the analytical justification of the algorithm and presents a simple example to show how it works in practice.

Moreover, hints are given about how this algorithm could be coupled to a general simulation system for calculation of transients, such as ATP/EMTP.

2.- The concept

The easy way to include in a general program for the calculation of transients, such as the ATP/EMTP family, an element with the complex non linear behaviour of ferromagnetic hysteresis, is to approach to it as if it were a controlled source.

The model of ferromagnetic hysteresis presented in this paper, named FERROHYS, is developed considering the inductor as a current controlled voltage source, with the sign convention of Fig. 1.

The final formulation of FERROHYS is then easily translated into a working element of ATP/EMTP using the auxiliary module MODELS.

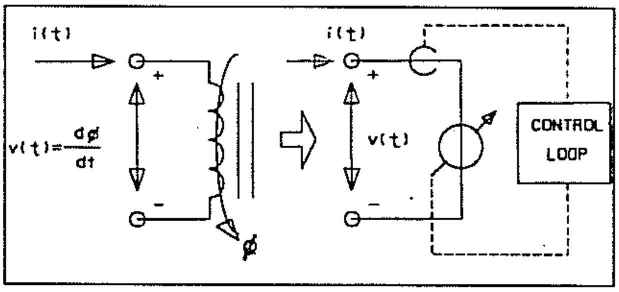

Fig. 1 - Concept of a current controlled voltage source equivalent to an inductor with magnetic hysteresis.

3.- The mathematical model

To develop a mathematical formulation for the proposed "control loop" of the equivalent voltage source, a conceptual model of the behaviour of the ferromagnetic material is needed, so we can relate the linked flux ϕ of the inductance vs. the current i trough it.

The conceptual model on which FERROHYS is based was presented in [5], in terms of magnetization M or flux density B vs. field strength H. A brief description is given in the Appendix.

Recalling that ϕ - B = μo (M + H) and i - H, the same basic mathematical model can be used for the ϕ vs. i relationship.

Accordingly, a general expression for ϕ as a function of i is written.

$$\phi(i) = \eta(i,\underline{h}).G(i)-[1-\eta(i;\underline{h})].G(-i) \qquad (1)$$

where G(i) is a non linear function, characteristic of the model, defined in the range $-\infty \le i \le +\infty$.

$\eta(i;\underline{h})$ is a distribution-like function which describes the actual "magnetic state" of the material, defined in the range $-i_{sat} \le i \le i_{sat}$.

$\underline{h}$ is a vector of the n relevant past values of i(t) which define the "magnetic history" at the actual time t.

i_{sat} is the value of $|i|$ for which "hysteresis saturation" is reached.

Note that the conventional i_{sat} of "hysteresis saturation" is such that in practice the ϕ vs. i curve for $|i| > i_{sat}$ is reversible, although it may still be non linear.

The voltage accross the inductance for a changing i(t) (and ϕ) is thus given by

$$v = \frac{d\phi}{dt} = \frac{d\phi}{di} \cdot \frac{di}{dt} = L(i)\,\frac{di}{dt} \qquad (2)$$

with

$$L(i) = \eta'(i;\underline{h}).\beta(i)+\eta(i;\underline{h}).\beta'(i)+\alpha(i) \qquad (3)$$

where

$$\eta' = \frac{d\eta}{di} \quad ; \quad \beta = G(i)+G(-i)$$

$$\alpha = G'(-i) \quad ; \quad \beta' = G'(i)-G'(-i)$$

$$G' = \frac{dG}{di}$$

Being i a function of t, so will L(i) be. Thus, multiplication of (3) by di/dt makes

$$v = \beta\,\frac{d\eta}{dt} + \eta\,\frac{d\beta}{dt} + \alpha\,\frac{di}{dt} \qquad (4)$$

This is a dynamical formulation of the model FERROHYS. It can be represented by the block diagram of Fig. 2. Note that the set of initial values $\phi_0 = 0$, $\eta_0 = 0{,}5$, $\beta_0 = 2G(0)$, corresponds to the basic assumption of an unmagnetized, "virgin", material at $t = -\infty$. The "magnetic history" of the material at t = 0 should be given by the initial values of the vector $\underline{h}(0) = (h_1, h_2, ..., h_n)$.

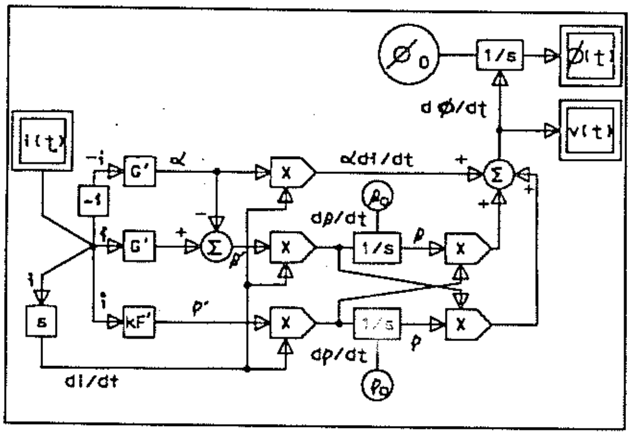

Fig. 2 - Block diagram of the model FERROHYS formulated as a dynamical process.

Notes: s is the Laplace variable.
G' and F' are non linear functions of i.
K is a non dimensional discrete function of i and $\underline{h}$.

3.1.- Characteristic functions

The model described is based on two characteristic functions, F(i) and G(i), and their derivatives, F' = dF/di and G' = dG/di.

G(i) is a form function, equivalent to Ψ(H) defined in the Appendix.

F(i) is the (cumulated) distribution function of the fictitious "coercitivities" of the idealized elements in the ferromagnetic material, which range between i = 0 and $i = i_{sat}$.

Hence it is easily deduced that we will have $\eta(i;\underline{h}) = F(i)$ when $\underline{h} = (-i_{sat})$, being n = 1, and i es steadely increasing from $i = -i_{sat}$ up to $i = i_{sat}$.

Both characteristic functions can be obtained from the information provided by the conventional limit hysteresis cycle (as in Type 96 of ATP/EMTP) and the curve of "reversible inductance" $L_{rev}(i)$.

Fig. 3 shows the basic concepts in grafical form.

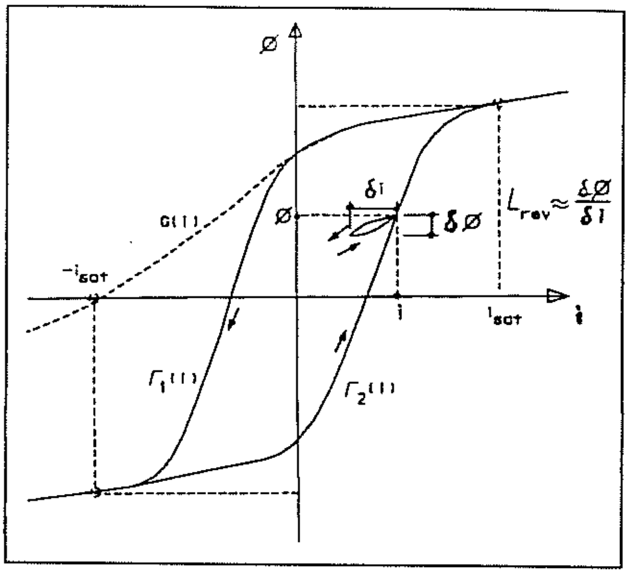

Fig. 3

Let $\Gamma_1(i)$ and $\Gamma_2(i)$ be, respectively, the upper and lower branches of the limit hysteresis cycle on the ϕ vs. i plane, defined for $-\infty \le i \le +\infty$ (note that $\Gamma_2(i) = -\Gamma_1(-i)$ and $\Gamma_1(i) \equiv \Gamma_2(i)$ for $|i| \ge i_{sat}$). And let $L_{rev}(i)$ be the curve of the "reversible inductance", defined as the limit of the ratio $\delta\phi/\delta i$ for a vanishingly small reduction of excitation, $\delta i < 0$, from a "working point" on Γ_2 with $0 \le i \le i_{sat}$.

It is shown in [5] that $G(i) \equiv \Gamma_1(i)$ for $i \ge 0$ and $G(i) = \Gamma 1(i_{sat}) - \Gamma_1(-i)$ for $i \le -i_{sat}$, being F(i) = 0 for $i \le 0$ and F(i) = 1 for $i \ge i_{sat}$. Moreover, G(i) in the range $-i_{sat} \le i \le 0$ and F(i) in the range $0 \le i \le i_{sat}$ can be calculated from the set of equations

$$\Gamma_2(i) = F(i).\Gamma_1(i) - [1 - F(i)].G(-i) \qquad (5)$$

$$L_{rev}(i) = F(i).\Gamma_1'(i) - [1 - F(i)].G'(-i) \qquad (6)$$

where $0 \le i \le i_{sat}$; G' = dG/di ; $\Gamma'_1 = d\Gamma 1/di$

By combining (5) and (6), F is eliminated and a first order differential equation is formulated for G and G', which can be solved numerically. Once G and G' are known, F is calculated from (5). Hence

$$F(i) = \frac{\Gamma_2(i) + G(-i)}{\Gamma_1(i) + G(-i)} \; ; \text{ for } 0 \le i \le i_{sat} \qquad (7)$$

We finally find the derivarive F'(i) = dF/di.

3.2.- The dynamical "history function" $K(i;\underline{h})$

The actual state of the magnetic material at time t will be described by a distribution-like function $\eta_t(i)$ which gives the fraction of unit volume having elements for which the next transition jump will occur at an excitation greater or equal than i.

The function $\eta_t(i)$ changes with t, but it is, an any time, completely determined by the distribution function $F(i_{cp})$, of the elemental coercitivities i_{cp}, and the magnetizing history of the material.

The magnetizing history, on the other hand, is given by the sequence of absolute extreme values of i(t) recorded between all pairs of succesive zero crossings, from actual t_0 backwards into the past.

Of this sequence, it is only relevant the unique subset of the n most recent extreme values $h_1, h_2, \ldots, h_n$, occurred at times $t_1 < t_2 < \ldots < t_n \leq t_0$, which fullfil the conditions

$$\text{abs}\,(h_{j+1}) < \text{abs}\,(h_j) \tag{8}$$

$$\text{sign}\,(h_{j+1}) = \text{sign}\,(h_j) \quad ; \quad j = 1, \ldots, n-1$$

These conditions imply that h_1, the "oldest" extreme value retained, is the highest absolute value ever reached by i(t), and that intermediate values of the original sequence of extremes which do not fit the stated conditions (8), should be dropped out.

However, if at any time is $|i(t)| \geq i_{sat}$, it should be set $h_1 = i_{sat} * \text{sign}(i(t))$ and $n = 1$, hence restarting the magnetizing history from then on. Also, when the material is in its "virgin" state it has "no history", therefore $h_1 = 0$, $n = 1$.

Hence, the "history" will be given by the vector $\underline{h} = (h_1, h_2, \ldots, h_n)$ whose dimension n may be changing from time to time.

At any time t_0, three typical situations may occur regarding the actual evolution of i(t) in reference to the history $\underline{h}$; they are:

(a) i(t) just made a zero crossing, changing sign, and it is increasing in magnitude, heading towards the next extreme value, but it has not yet surpased the absolute value of the previous one, h_n. In that case

$$\left.\frac{d\eta}{dt}\right]_{t_0} = F'\,(|i_0|)\cdot\left.\frac{di}{dt}\right]_{t_0} \quad ; \; i_0 = i(t_0)$$

(b) i(t) is increasing in magnitude as in (a), but its absolute value is now greater than any previous one, that is $\underline{h} \equiv (i_0)$, being n = 1. In that case

$$\left.\frac{d\eta}{dt}\right]_{t_0} = \frac{1}{2}\cdot F'\,(|i_0|)\cdot\left.\frac{di}{dt}\right]_{t_0} \quad ; \; i_0 = i(t_0)$$

(c) i(t) is decreasing in absolute value, having just gone through an extreme value of such a magnitude that it is now the latest h_n. In that case

$$\left.\frac{d\eta}{dt}\right]_{t_0} = 0 \quad ; \quad i_0 = i(t_0)$$

Any actual situation for i(t) will fall into one of these three cases. Hence, to define in a general way the instantaneous behaviour of the dynamical variable $\eta(i;\underline{h})$, we can write

$$\frac{d\eta}{dt} = K(i;\underline{h})\;.\;F'(|i|)\cdot\frac{di}{dt} \tag{9}$$

where $K(i,\underline{h})$ will take one of three numerical values: 1 in case (a), ½ in case (b) and 0 in case (c).
Fig. 4 shows the flow diagram representing the algorithm for the ongoing calculation of $K(i,\underline{h})$ and the "refreshing" of the components of the history vector $\underline{h}$, as well as its dimension n.

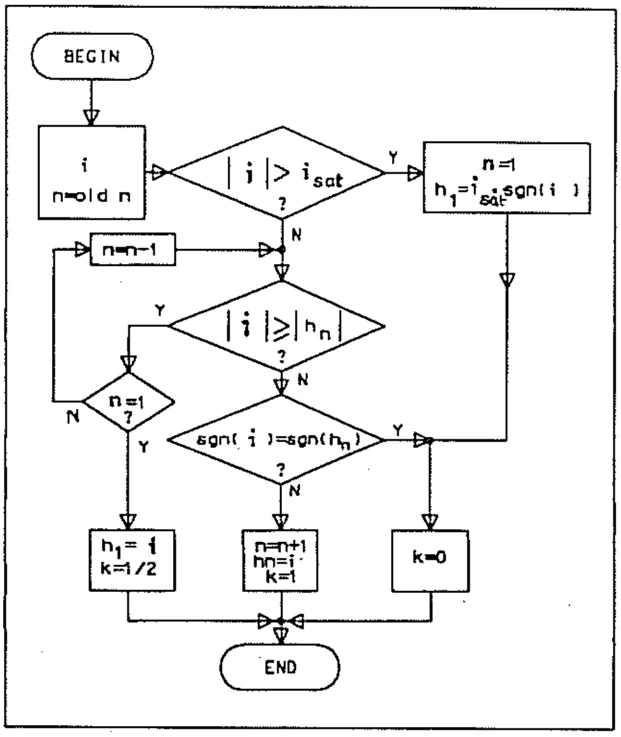

Fig. 4

3.3.- Important features

The model FERROHYS gives a very good representation of most of the important phenomenae of ferromagnetic hysteresis, namely: nonlinearity, irreversibility, saturation, partial remanence, "memory".

There reamins, however, some particular features which turn out to be represented in a simplified form:

(i) the "virgin" magnetization curve is the mean value of the upper and lower branches of the limit hysteresis cicle.

(ii) the remanent incremental inductance (due to the so called remanent permeability) is constant, at any (partial) value of remanence.

(iii) small reductions of magnetization (to be associated with the incremental permeability) appear as entierely reversible as long as an inversion of sign is not reached.

All these simplified features are however close enough to the real behaviour of most practical ferromagnetic materials.

Moreover, eddy current effects are not taken into account by FERROHYS, so they are to be simulated by some additional component, e.g. a suitable resistante in parallel [4].

4.- An Example

To show the performance of FERROHYS, a very simple example has been set up: a modulated alternating current source directely connected to an inductor with unity "turns times magnetic-material-cross-section" and unity "turns per magnetic-circuit-length", making thus $\phi = B$ and $i = H$.

The magnetic material is taken to be 4-79 Permalloy, whose basic experimental data can be found in [3], also referred to in [5].

For the supposed inductor with this material, the functions F(i) and G(i) are equivalent, respectively, to $v_2(H)$ and $\Psi(H)$ given in [5] (see Appendix). The material is considered virgin at t = 0.

The current source for this test set up is

$$i(t) = 0,2\ [\sin(2\pi.2,5\ t)]^2 \sin(2\pi.50\ t)$$

Hence, the inductor will be excited by an alternating field which first increases in amplitude up to 0,2 A, and then decreases down to 0.

This test case was made using the version ATP7 of EMTP, under license of use given by the Canadian-American Users Group through the CAUE (Comité Argentino de Usuarios del EMTP).

The time functions of current through the inductor i(t) and voltage accross it v(t) are shown in Fig. 5.

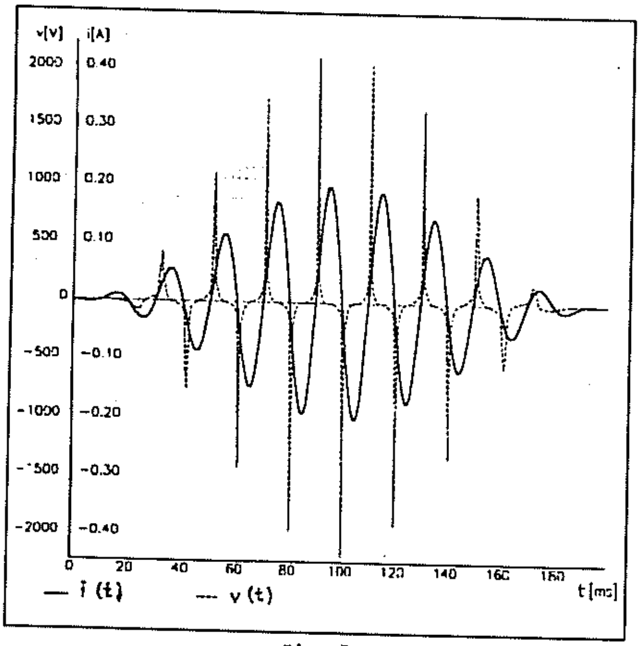

Fig. 5

Those graphs give a clear illustration of the fair representation of ferromagnetic hysteresis which is obtained with FERROHYS.

5.- Conclusions

5.1.- A practical algorithm for the computation of ferromagnetic hysteresis has been presented, useful for simulation of transients in electrical circuits with a general computer program such as ATP/EMTP.

5.2.- The algorithm was programmed within the module MODELS of ATP/EMTP and its ability shown with a simple example.

5.3.- Further work should be done for a better representarion of some features of ferromagnetic behaviour in general: eddy current effects, "local" hysteresis for samll incremental changes of excitation.

7.- References

[1] - Leuven EMTP Center: "Alternative Transients Program Rule Book". Leuven, 1987.

[2] - G. Biorci and D. Pescetti: "Analitycal theory of the behaviour of ferromagnetic materials". Il Nuovo Cimento, vol. VII, Nº 6, 1968, pp. 829-842.

[3] - R.M. Bozorth: "Ferromagnetism" (7th. printing). D. Van Nostrand Co., Inc., Princeton, New Jersey.

[4] - Germay N., et al.: "Review of Ferro-resonance Phenomena in High Voltage Power System and Presentation of a Voltage Transformer Model for Predetermining them". CIGRE, Report 33-18, 1976.

[5] - Riubrugent J., et al.: "A Mathematical Model of Ferromagnetic Behaviour Suitable for Simulation of Transient Phenomena in Electrical Power Systems". CBmag'95, Florianópolis, Brasil, may 14-17, 1995.

APPENDIX

Conceptual model of hysteresis in ferromagnetic materials

It is not intended that the model to be described is thought of as being related to any physical explanation; it is rather a "thinking device" used for developing a useful calculation procedure.

i) The ferromagnetic material is conceived as an aggregate of "bi-stable elements", with an idealized hysteresis loop such as is shown in Fig. A-1.

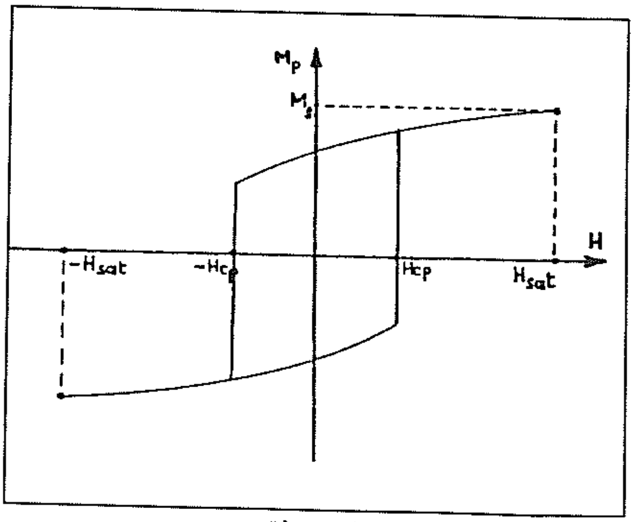

Fig. A-1

ii) The upper and lower branches of the hysteresis loop Mp vs. H of a given element p are described by the functional relationship.

$$Mp\ (H) = \begin{cases} \Psi(H) \text{ for } -Hcp < H \text{ (upper branch)} \\ -\Psi(-H) \text{ for } H < Hcp \text{ (lower branch)} \end{cases} \qquad (1)$$

iii) The function $\Psi(h)$ is <u>the same for all elements</u>, being a characteristic of the material, positive and monotonically rising in the range $-H_{sat} < h < +H_{sat}$, where H_{sat} is the value of field intensity at which saturation of the material is assumed.

iv) The transition "jumps" between the upper and lower branches occur symmetrically at values of H = ±Hcp, the caracteristic "coercitivity" of element p. It is precisely the parameter Hcp which distinguishes the particular behaviour of element p from that of the others.

v) The infinitesimal elements have a uniform, homogeneous distribution in space, such that the elements contained in any arbitrarily small volume of material will always have the same distribution function of the characteristic parameter Hcp.

vi) Since the relationship between M_p and H is in part double valued, at any given external exciting field H the <u>state</u> of a given element p could be such that the corresponding value of M_p (H) may fall onto either of the branches of the hysteresis loop if -Hcp < H < Hcp, depending on the past evolution of H (history).

vii) The macroscopic magnetization M(H) of the bulk of ferromagnetic material will thus be the net value of the added contribution of the elements which, for the given H, are in the "up state" (Mp = Ψ(H), with the same polarity than H), substracting the contribution of those which are in the "down state" (M_p = - Ψ(-H), opposing the polarity of H).

The postulated conceptual model may be described mathematically as follows.

Let us considere, at time t, the relative quantity of elements contained in a unit volume for which the "next jump" in the hysteresis loop (trasition fron one of the branches to the other) would occur at a value of H greater than a given h. The "actual state" of the material at time t will be described by the distribution function $v_t(h)$ of the fraction of unit volume having elements for which the "next jump" is greater o equal than h.

Since, because of changes of H, some elements will eventually switch " state" making their "next jump" to change sign, the distribution function $v_t(h)$ will be changing to follow the evolution of H(t).

Of course, $v_t(h)$ will always be a monotonically increasing function, fron $v_t(h) = 0$, for $h \leq -H_{sat}$, up to $v_t(h) = 1$, for $h \geq H_{sat}$.

Known the above defined $v_t(h)$ as well as $\Psi(h)$, the macroscopic net value of magnetization M (H,t) wil be given by the general formula:

$$M(H,t) = v_t(H).\Psi(H) - [1-v_t(H)].\Psi(-H) \qquad (2)$$

where H is the applied external field, assumed to be positive.

Permalloy 4-79 was taken as an example of typical ferromagnetic material. Fig. A-2 shows the <u>limit hysteresis cycle</u> with some "nearly reversible" minor loops from which for the <u>reversible permeability</u> curve can be estimated. From these data the characterictic functions $\Psi(H)$ and v_0 can be calculated.

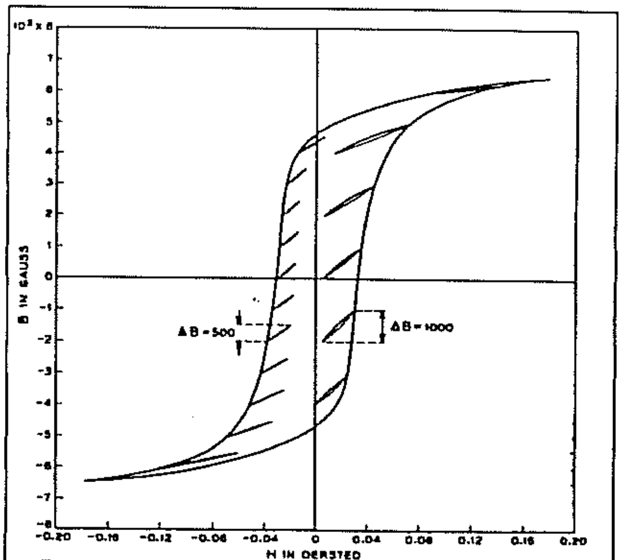

Fig. 2.-Minor and major hysteresis loops of 4-79 Permalloy. For the minor loops ΔB is constant at 500 (left side) or 1000 (right side) gauss. [3]

Fig. A-2

Fig A-3 shows the result of direct application of the model presented for a process of alternating excitation H with increasing amplitude, represented in the B vs. H plane.

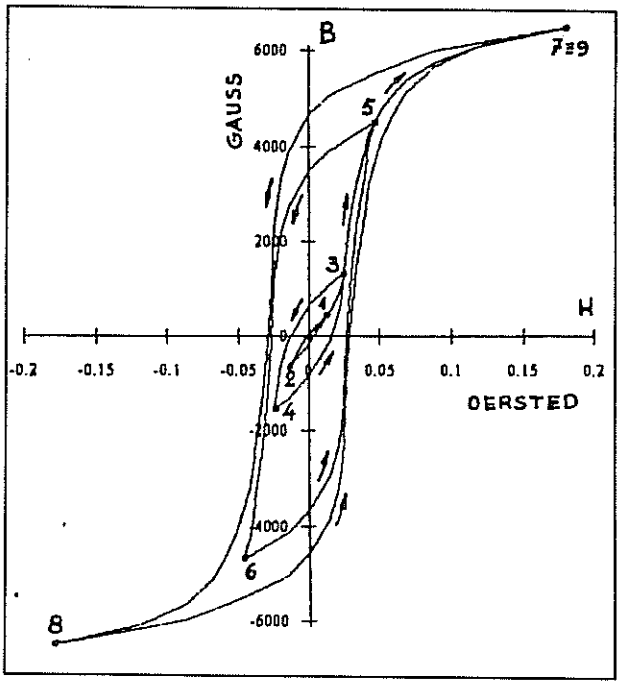

Fig. A-3